

An Analysis of the Dilemma of Protagoras and Euathlus and the Librarian's Paradox

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Dilemmas and paradoxes, such as those we will examine here, which might conceivably occur in real-life situations, often capture the attention and imagination in a way that their purely symbolic forms could not. Consequently, students may be surprised to find themselves solving problems which they would not have even attempted had those problems been presented in symbolic form. As such, I think attention should be given to these type of argument and to the variety and complexity of the thinking skills needed to understand and solve them. This paper will follow a simple format — divided into three main parts and followed by a conclusion. Parts I and II concern the dilemma of Protagoras and Euathlus and each will make use of a dialogue. Part I will identify the source of the contradictory conclusions involved in the dilemma. Part II will deal with the practical matter of deciding the case. In Part III, the librarian's paradox will be discussed. Mention will be made of its relationship to the Protagoras/Euathlus dilemma and to the liar paradox. In the conclusion I will present some final thoughts concerning the role of situational dilemmas and paradoxes, such as those presented here, in the teaching of logic and philosophy, and some advantages they may have over their mathematical counterparts, especially in introductory courses.

It should be noted that paradoxes and dilemmas are two closely related types of argument. Dilemmas are often paradoxical. In addition, a paradox can be formulated as a dilemma, and this may facilitate the understanding of the problem. The dilemma of Protagoras and Euathlus, and the librarian's paradox can be seen as both paradoxical and dilemmatic.

Part I

The dilemma of Protagoras and Euathlus¹ arose from a lawsuit brought to court in Ancient Greece in the fifth century B.C. An agreement had been made between the sophist Protagoras and his student Euathlus concerning the future payment of tuition money. Copi relates the details of the agreement as

follows:

Eulathus wanted to become a lawyer, but, not being able to pay the required tuition, he made an arrangement according to which Protagoras would teach him but not receive payment until Eulathus won his first case. When Eulathus finished his course of study, he delayed going into practice. Tired of waiting for his money, Protagoras brought suit against his former pupil for the tuition money that was owed. ...Eulathus decided to plead his own case in court. (208)

Protagoras presented his case in the form of a simple constructive dilemma, which at first, appeared to be a sound argument: if I win, he has to pay me — that is the decision of the court. If I lose, he has won his first case, and therefore has to pay me — that is the stipulation of the contract. I will either win or lose. Therefore, either way, he must pay me.

Euathlus defended himself with a counter-dilemma that had the same strengths and/or weaknesses as the case put forth by his teacher: If I win, I do not have to pay — that is the decision of the court. If I lose, I will not yet have won my first case, and therefore, I will not have to pay — that is the stipulation of the contract. I will either win or lose. Either way, I do not have to pay.

Copi puts forth the question, “had you been the judge, how would you have decided?” (209)

Before we can address this question, we must discover the source of these contradictory conclusions arrived at by the teacher and student. At first it may seem a perplexing task. But upon a more careful examination, it is not difficult to identify the problem.²

Both the teacher and the student are inconsistent in their positions as to which has supreme authority — the court or the contract. When it serves their interests — supports their position — each will assign supreme authority alternately to both the court and the contract, which conveniently ignoring the authority or binding force of the other. They contradict themselves.

The teacher’s position, once again, with hypothetical cross-examination:

Teacher: Either I will or I lose. Either way, he has to pay me. If I lose, he has won his first case, and therefore must pay me — under the terms of the contract.

Cross-examiner: Yes, but the fact that he has won means that the court has decided that he does not have to pay.

So you must believe that the contract carries more legal weight than does the court.

Teacher: As I was saying, if he wins, he has to pay me. And if I win, he must pay me — by order of the court.

Cross-examiner: Ah, but what about the contract still in force — and you yourself understand it to be still in force, for you just used it to support your position. According to the contract, since the student will have lost, he will not yet have won his first case, and therefore would not have to pay you. So now, you are saying that the court carries more weight than the contract. Yet you just gave the opinion that the contract carries more weight than the court. You must realize, as a sophist, that this is a direct contradiction. So your position is that when it's convenient for you to have the court supreme, the court is supreme, and when it is convenient for you to have the contract supreme, the contract is supreme. Is this your argument? Protagoras, you cannot have it both ways. Please explain yourself.

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Of course, the student could be cross examined in the same fashion. However, we know the teacher was intentionally trying to mislead the court with his “cleverly devised sophism” (Gellius 1: 409). The student adopted the same for only to defend himself. This type of argument, termed “convertible,” has been referred to by Gellius as “by far the most fallacious,” “among fallacious arguments.” Such arguments “may be turned in the opposite direction and used against the one who has offered it, and is equally strong for both sides of the question.” (Gellius 1: 405)

Part II

Though we have identified the difficulty as arising from the contradictory premises of “the court has precedence over the contract” and “the contract has precedence over the court,” this does not decide the case. The arguments cancel each other out. This leaves the teacher and the student on even footing.

Once again, the terms of the agreement: “Protagoras would teach him but not receive payment until Euathlus won his first case.” Therefore, looking only at what is stated, Euathlus does not have to pay until he has won his first case. At the time Euathlus is taken to court by his former teacher, he has not yet won his first case, and therefore Protagoras has no grounds to sue him that are

rooted in any part of the contract. Protagoras is tired of waiting and therefore takes the student to court. Is there any clause in the agreement that says the student must pay him when he wins his first case or when Protagoras grows tired of waiting — whichever comes first? There is not. Therefore, the student does not have to pay. The teacher will just have to wait and hope for the best.

Some might disagree with this decision. Perhaps there are those who would hold that the contract implicitly requires that the student actually practice law because that is the only way the teacher can be paid back; and therefore, that his failure to practice law constitutes breaking the contract, and, since he has broken the contract, he owes his teacher the tuition. But in this line of argument, these supporters of Protagoras would soon run into difficulties from which they could not extricate themselves, and interestingly, the teacher himself did not use this defense.

Let us look at a dialogue in which a man holding this position is cross-examined by one who wishes to refute his argument. For this dialogue, we will assume the teacher waited one year before taking the student to court. Were I the cross-examiner, I would begin by restating the terms of the agreement, as follows:

Cross-examiner: "Protagoras would teach him but not receive payment until Eulathus won his first case." So it is your position that somewhere in this brief contract, there is an implied requirement that the student must practice as a lawyer, because that is the only way the teacher can be repaid — something that can be ascertained from the contract itself? And if he does not practice as a lawyer, then he is in violation of the agreement? And since he has broken the contract with his teacher, he owes him the tuition money?

Supporter of Protagoras: Yes.

Cross-examiner: So, as long as he practices as a lawyer at some point in his life time, he is following the agreement — even if he begins in his old age.

Supporter of P: No — for then his teacher would be dead by the time the student has to pay the money. He cannot delay indefinitely.

Cross-examiner: Ah, so now you are saying that somewhere, implied in this agreement, is the requirement, not only that he must practice as a lawyer, but that he must practice as a lawyer within his teacher's lifetime.

Supporter of P: Yes, of course.

Cross-examiner: But surely you know that both

teacher and student are alive and well. Therefore, the student, even considering these so-called implied requirements, is now in violation of his agreement. Yet you still feel that the teacher has the right to sue?

Supporter of P: Yes.

Cross-examiner: So now, your position must be that somewhere, implied in this agreement, are stipulations that the student must practice as a lawyer, within the lifetime of his teacher, and must begin his practice by a certain date within the teacher's lifetime. And yet, there is no such specific date mentioned in the agreement.

Supporter of P: Well, he must begin promptly — within a reasonable amount of time.

Cross-examiner: A reasonable amount of time? So, if he began practicing the day after graduation, he would keep his part of the agreement?

Supporter of P: Yes, exactly.

Cross-examiner: But, since it has been a year since the student graduated, and he has not yet begun his practice, you would say he was definitely in violation of the agreement.

Supporter of P: Yes.

Cross-examiner: Well, what of the student decided to take a respite after completing his studies. If he decided to wait three weeks before starting his practice, would he be in violation of the agreement?

Supporter of P: Well, no.

Cross-examiner: Why?

Supporter of P: Because that is a reasonable amount of time.

Cross-examiner: And what if he waited six months — would he be in violation of the agreement?

Supporter of P: I suppose not, for then his teacher would have brought him to court after six months.

Cross-examiner: So he would not be in violation of the agreement. Now what if his teacher thought it perfectly acceptable for him to take time off to travel and see the world — to take two years or three, or ten. According to your line of reasoning, as long as his teacher did not object, the student still would not violate his agreement. So according to your argument of implied stipulations, whether or not the student is in violation of his agreement, all depends on the teacher's subjective view of waiting. And, how tired must he be before the student is in violation of his agreement? And how are we to know objectively when the teacher has reached this threshold of tiredness? Do you not see the absurdity of this

mode of argument?

Supporter of P: (no reply)

Cross-examiner: Now surely you will listen to reason. We can imagine a teacher having a variety of attitudes or reactions to such a situation. What other implications might we "discover" in this agreement, if we had cause to look for them? We can imagine a case in which the teacher would still take the student to court even if he did pay him the money, but, decided not to practice law. Perhaps such a teacher would regard his time as more important than money. He taught, not just to collect a fee, but to form the mind of a young lawyer for the next generation, to carry on the art of argument. In that case, even if the teacher received his tuition, he might still sue him for not practicing law, rendering all his teaching useless — intending to penalize him in some way. Taking this position, the teacher would consider the student to have violated his agreement even if he paid the tuition, because he was not practicing as a lawyer. Or we can imagine the teacher having a completely different attitude. He does not expect him to pay the tuition at all. Since the student found the law profession unsuitable to him, for whatever reason, he should not have to pay for it. Since the agreement was that he should pay as soon as he won his first case, we might infer that he should pay because he is beginning to receive the intended benefit from the teacher's instruction — a successful law practice and still neglected to pay his tuition. Or, we might infer that the contract is set up this way so that he will pay the teacher when he can afford to. Winning his first case will bring him financial rewards. Therefore, we might conclude that he should pay him back as soon as he can afford to. Under this interpretation, if he should receive a large inheritance the day of graduation, he would have to pay then, even though he has not yet won a case. Could not all these scenarios be compatible with the contract: "Protagoras would teach him but not receive payment until Eulathus won his first case." Do you not see now that these so-called implied requirements are not in fact based on any part of the contract?

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As the preceding dialogue illustrates, we could suggest any number of meanings that might have been intended. But it cannot be demonstrated that they are a necessary part of the contract. We must go by the actual wording of the contract itself. As we have seen, when the teacher takes him to court, the student has not even had his first case yet, much less won it. So the student is

not in violation of his agreement. Therefore, the teacher had no grounds to sue him. As such, the teacher, by taking the student to court, is violating his part of the agreement in that he is trying to force the student to pay money which, under the terms of the contract, he does not owe. The fact that the student delays going into practice, though unfortunate for the teacher, is not relevant, for there is no stipulation in the agreement that the student must begin his practice immediately or, in fact, begin it at all. The teacher erred in making this assumption without specifically including it in the contract. Technically, even with such a stipulation, were the student determined not to pay the teacher, he could begin his practice immediately; but deliberately lose the case and then go into retirement, and therefore, he would not have to pay. The teacher made an unwise agreement. One certainly cannot assume that a student will ever work in the profession for which he has studied, much less that he should begin promptly. We can imagine the disastrous results for our modern day institutions of higher education if the tuition payment plans were arranged according to a similar contract. Though we may have sympathy for the teacher in his predicament, we cannot allow such emotions to obscure the issues at hand. We see that if the contract had contained a requirement that the student begin his law practice by a certain date, there would be no problem. The case would be easy to decide. It is quite apparent that the contract as stated is inadequate to protect the financial interests of the teacher. He should never have agreed to such terms. It is not the responsibility of the law to rescue people from the results of their own foolish decisions. It is my opinion that the teacher should accept his losses gracefully and regard the episode as a learning experience.

Part III

In a logical paradox two statements are incompatible or contradictory, yet are apparently both true. "Each is backed by an argument which seems correct" (Carney 147). A thorough analysis of the librarian's paradox reveals several interesting facets often ignored or overlooked in brief explanations or descriptions of the dilemma in which it is often dismissed as a mathematical enigma or an unlikely and irresolvable practical problem. The version with which I am acquainted follows.

In every branch library an index is made of the collection of books. Some of the librarians choose to include the title of the index itself in the index.

Others decide to leave it out. Duplicates of all the indexes are sent to the central library. The head librarian now wants to make two master indexes of the indexes, one for all those that include themselves, and another for all those that do not include themselves. It seems he has no trouble with the first, "The Master Index of All Indexes that Include Themselves," and includes the name of the master index in the master index, for he now has a complete listing of all the indexes that include themselves — the master index itself being one of them.

This first part of the librarian's task was not presented as problematic. However, we should note that technically, noting the sequence of events, as he takes up the pen to write in the name of the master index in the master index, he has no valid reason to do so, for at that moment or any moment previous, it was not an index that included itself. His decision to put in the name is based on the foreknowledge that it will be a true statement when it is completed. We see, looking at the final result, that this is the case. The inclusion of this title or any title in the index identifies or defines it as an index that includes itself. Translated into a proposition we have: "The Master Index of All Indexes that Include Themselves" is an index that includes itself. And, were we able to examine his book, we could easily verify that it was true. But before the act is accomplished, the librarian has decided to place the title of an index in the master index, which does not include itself, and therefore does not belong there, by definition of the type of index the master index will include. The librarian is thinking, "It does not include itself now, but will, once I have entered the title." The paradox is, that when the librarian first touches the pen to the paper, he is beginning to write a falsehood, but by the time he has finished, he has written a truism. One could say that in writing down the false statement, he simultaneously makes it true.

Putting the librarian's mental dilemma and decision aside and regarding only the written record, we might ponder whether or not in the physical act of writing in the title, there at some point occurs a split second of contradiction — an intermediate state — between true and false, or, whether there is simply a fine line between truth and falsity sweeping across the page as the librarian makes a false statement true simply by putting it down in ink. But lest I should stray too far from the subject, such considerations will not be taken any further here.

Now let us take a look at the second part of the problem — the part traditionally presented as paradoxical. The librarian begins to make an index of

all indexes that do not include themselves. But he ponders whether or not he should include the title of this master index in the index itself. He soon realizes that if he writes the title in, it will no longer be an index that does not include itself, and therefore will not belong in "The Master Index of All Indexes That Do Not Include Themselves." On the other hand, if he leaves it out, then it will be an index that does not include itself, and therefore will belong in the master index. Therefore he will not have a complete listing of all such indexes. The master index claims to be a complete listing, and therefore the index will not be what it is claimed to be. In simplified form: If he leaves the title out, he should put it in; and if he puts it in, he should leave it out.

The problem stated in dilemma form: If I include the title, it will no longer be an index that does not include itself, and therefore will not belong in the master index. If I leave it out, the listing will not be complete. I can either include the title, or leave it out. Therefore I will either have an incorrect entry, or an incomplete index.

Interestingly, this side was presented as the paradox while the other was to be presented as posing any such dilemma. It is the opposite paradox of the first. The difference is, that in the first case, the end result was a true statement with no remaining paradox. In this case however, the librarian, as he puts pen to paper in writing down the title, begins to write a true statement; but when he finishes, it is false. As a practical matter, what should the librarian do? Where I am in his place, I would conclude that including the title would be more objectionable than leaving it out. It seems that an error of omission is less critical. To include the title would be to define it as something it is not — a direct contradiction — a false statement. The index would then properly belong in the other volume, "Master Index of Indexes That Include Themselves." If it is left out, at least all the individual entries are correct and belong in the books. And, leaving the title out presents no problem as the book is being compiled, for it is not yet finished, and therefore, not part of the collection. Only when it is put on the shelf does the difficulty arise. For then, one book which meets the criterion for inclusion will be absent. A little explanatory text in the preface would suffice to eliminate the contradiction by slightly redefining what the master index will include, though it will not resolve the dilemma: "The Master Index of Indexes That Do Not Include Themselves" includes all such indexes excepting the Master Index itself.

There is no way to solve the paradox as stated. The selection of either

option results in a false claim. It is impossible to satisfy both conditions — that every index included is one that does not include itself, and that all such indexes be included.

Carney presents an explanation of the resolution of this type of paradox.³ From the assumption that it is possible to have an index of all indexes that do not include themselves, we are able to deduce a contradiction. Thus the assumption that there could be such an index is false. The difficulty is, that in specifying the criteria the master index must fulfill, “we unwittingly make them impossible, thereby eliminating the possibility” that such an index could exist. (148)

He also states that arguments, “which establish a surprising but true conclusion, are called veridical paradoxes” (149). Clearly then, the librarian’s paradox is veridical, for our analysis established the surprising but true conclusion that there could be no such index — one that includes all those, and only those, indexes that do not include themselves.

The problem in the librarian’s paradox deals with self-reference. To satisfy one of the two necessary conditions, the index would have to refer to itself and in so doing, makes it impossible to satisfy the second condition.

There is an element of self-reference in the case of Protagoras and Euathlus. What is the relationship between the case brought to court, and the case referred to in the contract (the first case Euathlus should win)? If we accept Protagoras’ position and Euathlus wins, then the case *about* the contract will be the same case referred to *in* the contract — the first case won by Euathlus. Then the case brought to court, since it refers to the contract, would be referring to itself, since it itself would be a component part of that contract — the first case Euathlus wins, mentioned in the contract.

The liar paradox is not veridical, yet it is related to the librarian’s paradox in that they both deal with self-reference. I will not attempt an in-depth discussion of the liar paradox, but briefly state: “This sentence is false.” Is the preceding sentence true or false? If it is true, then what it states must actually be the case, therefore it is in fact false. If it is false, then it is false that the sentence is false, then therefore it must be true. So if it is true, it is false; and if it is false, then it is true. (Carney 149-150)

We might gain some further insight by breaking the sentence down into its components (subject and predicate). “This sentence” is the subject, and “is false” is the predicate. But we see, that once the sentence is broken down into

its separate components, there is no longer any corresponding reality to which the subject can refer. The sentence no longer exists. It has been separated into two phrases. The phrase, "This sentence," is only meaningful if left in its original context. The sentence may be thought of as a self-contained closed system. It functions as its own subject, and is therefore indivisible if the subject is to have meaning. In writing the sentence, when we put down the phrase "This sentence," there is no existing sentence to which it refers; but when we finish, there is. We create the reality as we write the sentence.

The liar paradox is of great importance in 20th Century logic, for the inconsistencies found within it, as well as some of the solutions proposed to it, raise questions about or threaten traditional modes of reasoning (Carney 150) and fundamental theorems in math and logic (Anderson 10). For example, one solution involves rejecting all self-referential sentences. However, "the difficulty with such a stance is that some of the most profound arguments in logic involve self-reference," and not all self-reference leads to contradiction (Anderson 8).

Interestingly, one suggested solution to the liar paradox seems reminiscent of a solution offered by Carney to the Protagoras/Euathlus case:

The most commonly accepted "solution" to semantic paradoxes such as the liar paradox makes use of the "levels of language" distinction. ... The language used to talk about some other language is considered to be on a higher level than the language talked about. They require that sentences asserting the truth or falsity of a given sentence be placed in a language at least one level higher than the given sentence.⁴ (Kahane 314)

One possible solution is to argue that the trial is *about* the payment arrangement — whether the arrangement has been violated. Thus *this* case, the one being tried, should not be considered as one falling under the terms of the arrangement. Thus Protagoras' second conditional premise is false. (Carney 143)

In sum, language used to talk *about* another language is on a different level — a higher level — than the language talked about. As such, the sentence on the higher level would have supremacy over the sentence on the lower level. Therefore, self-referential sentences such as that of the liar paradox, which

function as both a lower and higher level of language, will be rejected as meaningless (Kahane 314). And, concerning the Protagoras/Euathlus dilemma, the case that is *about* the contract cannot be considered to qualify as the case referred to *in* the contract. So it seems we have come full circle in our discussion — from an ancient dilemma, to a problem of 20th century logic, and back again.

In Conclusion

The librarian's paradox, like the dilemma of Protagoras and Euathlus, may not be on the cutting edge of philosophical enquiry, but in this writer's opinion, they, and other similar, hypothetical cases have a particular contribution to make to the development of critical thinking. Because they are presented as practical dilemmas occurring in real-life situations, students may find these scenarios more interesting and easier to grasp than the more abstract theories behind them. To weave a story into and around the basic symbolic form does not compromise the underlying principles involved. It does make the study of logic more interesting, and less threatening. As stated in the introduction, many students would be astounded to see the symbolic formulas that correspond to problems they were able to solve in verbal form. And, with such knowledge, perhaps they will be less intimidated by symbolic logic. In addition, they contain situations about which students are not likely to have strong feelings. By using neutral topics in introductory studies in logic and philosophy, a student's thinking skills can be allowed to develop without the distracting and often destructive influence of emotionally charged topics. Hopefully, the student will have accepted certain logical procedures or rules by the time he or she is ready to apply these principles to other areas of philosophy, such as ethics or philosophy of religion, in which the student's emotions are apt to interfere. It is my hope that teachers will continue to make use of these time-honored and valuable tools.

Works Cited

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Notes

¹It appears there are several spellings of the name Euathlus. I have adopted the version used by Gellius, except in direct quotations.

²The solution I propose is quite similar to that offered by Lewis Carroll in his book, Symbolic Logic. However, my conclusions were arrived at independently, before I became aware of Carroll's work.

³Carney's explanation refers specifically to another paradox which, like that of the librarian, is veridical. I have applied the same form to the resolution of the librarian's paradox.

⁴Not all philosophers accept the levels of language solution.
(Kahane 315)